Algebra 2 // Topic 4 // Test Review A

1 $(4-6i)+(2-i)$

$4-6i+2-i$

$\boxed{6-7i}$

2 $(8+4i)+(6-2i)$

$8+4i+6-2i$

$\boxed{14+2i}$

3 $(7+3i)-(2-5i)$

$7+3i-2+5i$

$\boxed{5+8i}$

4 $(2-6i)-(3-2i)$

$2-6i-3+2i$

$\boxed{-1-4i}$

5 $6i(2i)$

$12\overset{-1}{i^2}$

$\boxed{-12}$

6 $(4+i)(4-i)$ FOIL

$16-\cancel{4i}+\cancel{4i}-\overset{-1}{i^2}$

$16-(-1)$

$\boxed{17}$

7 $(2+5i)(3-i)$

$6-2i+15i-5i^2$

$6+13i-5\overset{-1}{i^2}$

$6+13i-5(-1)$

$6+13i+5$

$\boxed{11+13i}$

8 $(6+2i)(1+5i)$

$6+30i+2i+10\overset{-1}{i^2}$

$6+32i+10(-1)$

$6+32i-10$

$\boxed{-4+32i}$

9 $\dfrac{5}{2+i}\cdot\dfrac{2-i}{2-i}$ FOIL

$\dfrac{10-5i}{4-\cancel{2i}+\cancel{2i}-\overset{-1}{i^2}}$

$\dfrac{10-5i}{4-(-1)}=\cancel{\dfrac{10-5i}{5}}$

$\dfrac{10}{5}-\dfrac{5}{5}i=\boxed{2-i}$

10 $\dfrac{6}{3-2i}\cdot\dfrac{3+2i}{3+2i}$ FOIL

$\dfrac{18+12i}{9+\cancel{6i}-\cancel{6i}-4\overset{-1}{i^2}}$

$\dfrac{18+12i}{9-4(-1)}=\dfrac{18+12i}{9+4}$

$\dfrac{18+12i}{13}=\boxed{\dfrac{18}{13}+\dfrac{12}{13}i}$

11 $\dfrac{3+3i}{2+5i}\cdot\dfrac{2-5i}{2-5i}$ FOIL

$\dfrac{6-15i+6i-15\overset{-1}{i^2}}{4-\cancel{10i}+\cancel{10i}-25\overset{-1}{i^2}}$

$\dfrac{6-9i-15(-1)}{4-25(-1)}$

$\dfrac{6-9i+15}{4+25}=\dfrac{21-9i}{29}$

$\boxed{\dfrac{21}{29}-\dfrac{9}{29}i}$

12 $\dfrac{4-2i}{1-4i}\cdot\dfrac{1+4i}{1+4i}$ FOIL

$\dfrac{4+16i-2i-8\overset{-1}{i^2}}{1+4i-4i-16\overset{-1}{i^2}}$

$\dfrac{4+14i-8(-1)}{1-16(-1)}$

$\dfrac{4+14i+8}{1+16}=\dfrac{12+14i}{17}$

For 1-12, you'll need to use $i^2=-1$.

13 i^2 $\boxed{-1}$

14 i^3 $\boxed{-i}$

15 i^4 $\boxed{1}$

16 i^5 $\boxed{i}$

17 i^{10} $\boxed{-1}$

18 i^{12} $\boxed{1}$

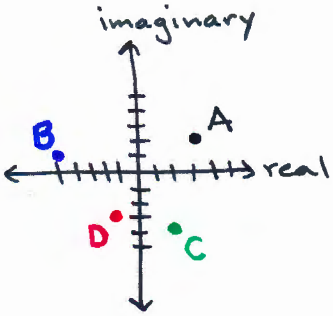
$$\boxed{\begin{aligned} i^1 &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$$

19 $3+2i$ A

20 $-5+i$ B

21 $2-4i$ C

22 $-1-3i$ D



23 $x^2+4x+8=0$

$\hookrightarrow a=1 \quad b=4 \quad c=8$

$$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(8)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-32}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$\sqrt{-16}$

$\sqrt{16}\sqrt{-1}$

$4i$

$$= \frac{-4 \pm 4i}{2}$$

$$= \frac{-4}{2} \pm \frac{4}{2}i \quad \text{reduce}$$

$$\boxed{-2 \pm 2i}$$

24 $3x^2 - \cancel{4} = 71$

$\quad\ \ \cancel{+4} \quad +4$

$$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{75}{3}$$

$$\sqrt{x^2} = \sqrt{25}$$

$$\boxed{x = \pm 5}$$

25 $x^2-2x-24=0$

$1\cdot 1$

$1 \quad 4$

$1 \quad -6$

$1 \quad -2 \quad -24$

$1\cdot -24 \qquad 4\cdot -6$

$-1\cdot 24 \qquad -4\cdot 6$

$2\cdot -12$

$-2\cdot 12$

$$(x+4)(x-6)=0$$

$x+\cancel{4}=0$

$\quad\ \cancel{-4} \quad -4$

$\boxed{x=-4}$

$x-\cancel{6}=0$

$\quad\ \cancel{+6} \quad +6$

$\boxed{x=6}$

When it asks to find the "solutions", then there are multiple ways to find them, examples #23, #24, and #25.

- to keep it simple, use X-factor if you cannot, then use quadratic formula.
- we have used more ways, like Completing the Square and simply square rooting.
- Recall. "x-intercepts" "solutions" "roots" "zeroes" all mean the exact same thing, so don't be confused.

26 $x^2 = 8x$

$-8x \quad -8x$

$x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0 \qquad x - 8 = 0$

$+8 \quad +8$

$\boxed{x=0} \qquad \boxed{x=8}$

27 $x^2 + 36 = 0$

$-36 \quad -36$

$\sqrt{x^2} = \sqrt{-36}$

$\boxed{x = \pm 6i}$

28 $2x^2 - x + 5 = 0$

$a=2 \quad b=-1 \quad c=5$

cannot x-factor → quadratic formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1-40}}{4} = \frac{1 \pm \sqrt{-39}}{4}$$

$$= \frac{1 \pm \sqrt{39}\, i}{4} = \boxed{\frac{1}{4} \pm \frac{\sqrt{39}}{4} i}$$

29 $2x^2 + 3x - 5 = 0$

1·2   1·−5   −1·5

2	3	−5

$(x-1)(2x+5) = 0$

$x - 1 = 0 \qquad 2x + 5 = 0$

$+1 \quad +1 \qquad -5 \quad -5$

$\boxed{x = 1} \qquad \frac{2x}{2} = \frac{-5}{2}$

$\boxed{x = -5/2}$

Make sure that when you get a complex answer that it is separated into the real and imaginary parts. Such as:

$$\frac{1}{4} \pm \frac{\sqrt{39}}{4} i$$

real → $\frac{1}{4}$ imaginary → $\frac{\sqrt{39}}{4} i$

30 ① The second number is 3 more than the first number. → $y = x + 3$

② The product of the two numbers is 9 more than their sum. → $xy = x + y + 9$

$y = x + 3$ — substitute

$xy = x + y + 9$

$x(x+3) = x + (x+3) + 9$

$x^2 + 3x = 2x + 12$

$-2x \; -12 \quad -2x \; -12$

$x^2 + x - 12 = 0$

1·1   1·−12   −1·12   2·−6   −2·6   3·−4   −3·4

1	1	−12

$(x-3)(x+4) = 0$

$x - 3 = 0 \qquad x + 4 = 0$

$+3 \quad +3 \qquad -4 \quad -4$

$x = 3 \qquad x = -4$

Using $y = x + 3$,

$y = 3 + 3 \rightarrow y = 6$

$(3, 6)$

$y = -4 + 3 \rightarrow y = -1$

$(-4, -1)$

$\boxed{\text{B.) } -4, -1}$

31 Zeros?

$x^2 - x - 12 = 0$

1·1 | 1·−12, −1·12, 2·−6, −2·6, 3·−4, −3·4

1 −4
1 3

$3 - 4$

1 −1 −12

$(x-4)(x+3)=0$

$x - \cancel{4} = 0$, $+\cancel{4}$ $+4$ → $\boxed{x = 4}$

$x + \cancel{3} = 0$, $-\cancel{3}$ -3 → $\boxed{x = -3}$

32 x-intercepts?

$3x^2 - 13x - 10$

1·3 | 1·−10, −1·10, 2·−5, −2·5

1 −5
3 2

$\frac{-15}{2}$

3 −13 −10

$(x-5)(3x+2)=0$

$x - \cancel{5} = 0$, $+\cancel{5}$ $+5$ → $\boxed{x = 5}$

$3x + \cancel{2} = 0$, $-\cancel{2}$ -2

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-2}{3}$ → $\boxed{x = -2/3}$

▷ $f(x)$ is the same as y. So,

$f(x) = x^2 - x - 12$

and

$y = x^2 - x - 12$

are considered to be the same.

33 Max value?

$y = -2(x-3)^2 + 4$

$(3, 4)$

$\boxed{4}$

34 Min value?

$y = 5(x+2)^2 + 3$

$(-2, 3)$

$\boxed{3}$

▷ **Max / Min Values**

A vertex at the top of a parabola is called a **maximum point**. The y-value is called the **max value**.

A vertex at the bottom of a parabola is called a **minimum point**. The y-value is called the **min value**.

35 Minimum Point

$y = x^2 + 6x - 4$

↳ $a=1$ $b=6$ $c=-4$

$x = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3$

$y = (-3)^2 + 6(-3) - 4$

$y = 9 - 18 - 4$

$y = -9 - 4 = -13$

$\boxed{(-3, -13)}$

36 Maximum Point

$y = -x^2 - 8x + 5$

↳ $a=-1$ $b=-8$ $c=5$

$x = \frac{-b}{2a} = \frac{--8}{2(-1)} = \frac{8}{-2}$

$x = -4$

$y = -(-4)^2 - 8(-4) + 5$

$y = -16 + 32 + 5$

$y = 16 + 5$

$y = 21$

$\boxed{(-4, 21)}$

↑ The max point or min point is just the vertex.

$\boxed{37}$ $y = 2x^2 - 20x + 1$ in $y = a(x-h)^2 + k$ form

$\hookrightarrow a = 2 \quad b = -20 \quad c = 1$

$x = \frac{-b}{2a} = \frac{--20}{2(2)} = \frac{20}{4} = 5$ $\downarrow h$

$y = 2(5)^2 - 20(5) + 1$

$= 2(25) - 100 + 1 = 50 - 100 + 1 = -50 + 1 = 49$ $\downarrow k$

$$\boxed{y = 2(x-5)^2 + 49}$$

OR $\rightarrow$ $2x^2 - 20x + 1 = 0$

$ -1 \quad -1$

$2x^2 - 20x + \boxed{50} = -1 + \boxed{50}$

$2(x^2 - 10x + 25) = 49$

$\frac{1}{2}$

$2(x - 5)^2 = 49 \quad \rightarrow \quad 2(x-5)^2 = 49$

$-49 \quad -49$

$2(x-5)^2 - 49 = 0$

$$\boxed{y = 2(x-5)^2 - 49}$$

Either way works! Use which ever way is easier for you. Actually, use which ever way you will make less mistakes.

$\boxed{38}$ $0 < a < 1$ $\boxed{\text{wider}}$

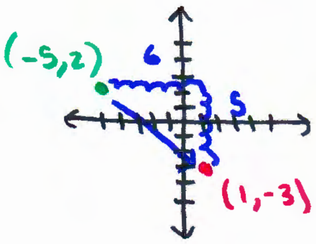
$\boxed{39}$ $a > 1$ $\boxed{\text{thinner}}$

$\boxed{40}$ $a < 0$ $\boxed{\text{downward facing}}$

41 Describe

$y=(x+5)^2+2$ to $y=(x-1)^2-3$

$\downarrow$ $(-5,2)$ $\qquad$ $\downarrow$ $(1,-3)$

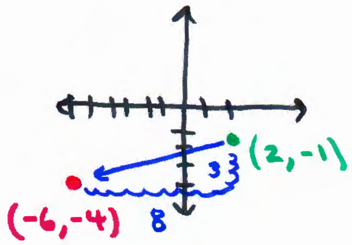


$\boxed{\text{6 to the right, 5 down}}$

42 Describe

$y=(x-2)^2-1$ to $y=(x+6)^2-4$

$\downarrow$ $(2,-1)$ $\qquad$ $\downarrow$ $(-6,-4)$



$\boxed{\text{Left 8, Down 3}}$

Be Careful. Make sure that you move from first point to the second.

43-45

A) vertices are maximums means both graphs are facing down.

B) graphs have same shape w/ different vertices mean that the "a" values are the same, but (h,k)'s are different.

C) graphs have different shapes w/ different vertices mean that the a, h, k are <u>all</u> different. There are no similarities.

D) One vertex has a max pt; one vertex has a min pt. That means one is facing up, while one is facing down. That means one a is positive and a is negative.

E) vertices are minimums mean both graphs are facing up. That means both a's are positive.

43 $y=-4(x+5)^2-3$ and $y=-4(x+5)^2+3$

$\hookrightarrow \underline{\underline{a=-4}}$ $h=-5$ $\underline{\underline{k=-3}}$ $\quad \hookrightarrow \underline{\underline{a=-4}}$ $h=-5$ $\underline{\underline{k=3}}$ $\qquad \boxed{B}$ and $\boxed{A}$

44 $y=2(x+8)^2+4$ and $y=-3(x+2)^2+1$

$\hookrightarrow a=2$ $h=-8$ $k=4$ $\quad \hookrightarrow a=-3$ $h=-2$ $k=1$ $\qquad \boxed{C}$ and $\boxed{D}$

45 $y=6(x+2)^2-7$ and $y=6(x+1)^2-3$

$\hookrightarrow a=6$ $h=-2$ $k=-7$ $\quad \hookrightarrow a=6$ $h=-1$ $k=-3$ $\qquad \boxed{B}$ and $\boxed{E}$

$\boxed{46}$ $f(x) = (x - b)^2 + c$ ... points up ... w/ mininum at $(-1, 6)$

$$f(x) = (x - \underset{h}{b})^2 + \underset{k}{c} \qquad (\underset{h}{-1}, \underset{k}{6})$$

So, $b = -1$ and $c = 6$.

$\boxed{47}$

$\boxed{48}$

$\boxed{49}$ $y=2(x+2)^2-4$

↳ $a=2$ $h=-2$ $k=-4$

$\boxed{1}$ up

$\boxed{2}$ Axis of Symmetry: $x=-2$

$\boxed{3}$ vertex: $(-2,-4)$

$\boxed{4}$

$$2(x+2)^2-4$$
$$2(x+2)(x+2)-4$$
$$2(x^2+2x+2x+4)-4$$
$$2(x^2+4x+4)-4$$
$$2x^2+8x+8-4$$
$$y=2x^2+8x+4$$

↳ $a=2$ $b=8$ $c=4$

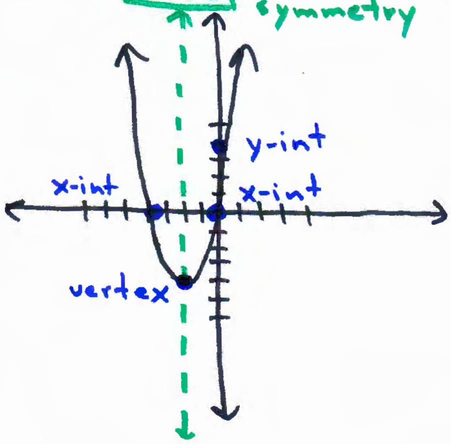
y-intercept: $(0,4)$

$\boxed{5}$ x-~~factor~~ → quadratic formula

$$x=\frac{-8\pm\sqrt{(8)^2-4(2)(4)}}{2(2)}$$
$$=\frac{-8\pm\sqrt{64-32}}{4}=\frac{-8\pm\sqrt{32}}{4}$$
$$=\frac{-8}{4}\pm\frac{4\sqrt{2}}{4}=-2\pm\sqrt{2}$$

$\sqrt{32}$
$\sqrt{16}\sqrt{2}$
$4\sqrt{2}$

$\boxed{x=-2}$ axis of symmetry



$\boxed{50}$ $y=-\frac{1}{2}(x-1)^2+3$

↳ $a=-\frac{1}{2}$ $h=1$ $k=3$

$\boxed{1}$ down

$\boxed{2}$ Axis of Symmetry: $x=1$

$\boxed{3}$ vertex: $(1,3)$

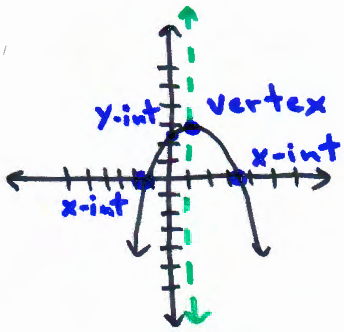
$\boxed{4}$

$$-\tfrac{1}{2}(x-1)(x-1)+3$$
$$-\tfrac{1}{2}(x^2-x-x+1)+3$$
$$-\tfrac{1}{2}(x^2-2x+1)+3$$
$$-\tfrac{1}{2}x^2+x-\tfrac{1}{2}+3 \quad \to -\tfrac{1}{2}+3=-\tfrac{1}{2}+\tfrac{6}{2}$$
$$-\tfrac{1}{2}x^2+x+\tfrac{5}{2} \qquad\qquad =\tfrac{5}{2}$$

↳ $a=-\frac{1}{2}$ $b=1$ $c=\frac{5}{2}$ $(0,\frac{5}{2})$

$\boxed{5}$ x-~~factor~~ or quadratic formula

$$x=\frac{-1\pm\sqrt{1^2-4(-\frac{1}{2})(\frac{5}{2})}}{2(-\frac{1}{2})} \quad \to -4(-\tfrac{1}{2})(\tfrac{5}{2}) \quad 2(\tfrac{5}{2}) \quad 5$$
$$=\frac{-1\pm\sqrt{1+5}}{-1}=\frac{-1}{-1}\pm\frac{\sqrt{6}}{-1}$$
$$=1\pm\sqrt{6}$$



$\boxed{x=1}$ Axis of Symmetry